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Flows with concentrated vorticity: a report on EUROMECH 41

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The 41st EUROMECH Colloquium on flows with concentrated vorticity was held in Norwich from 17 to 21 September 1973. There were sixty-five participants from nine countries and the author was the chairman of the organizing committee.

1. Introduction

The description by Küchemann (1965), in his report on the IUTAM Symposium, of vortex motions as 'the sinews and muscles' of fluid flows is no less apt today. Evidence for this is provided by the interest shown in the meeting under review and by the diversity of the papers presented. The subjects discussed divide into the following fairly broad classes: (i) vortex sheets associated with lifting surfaces; (ii) stability and decay of vortices; (iii) atmospheric vortices; (iv) vortex rings. This subdivision is adhered to in the sections below. Vortex shedding from bluff bodies was not included, as a recent Colloquium was devoted entirely to this topic (Mair & Maull 1971).

A most appropriate introduction to the Colloquium was provided by H. Werlé in the form of a spectacular colour film of flow visualizations obtained in the ONERA water tunnel and hydraulic tank. Most of the phenomena which formed the basis for the subsequent sessions were illustrated in this film.

2. Vortex sheets associated with lifting surfaces

Much of the recent work on leading-edge separation from slender wings stems from the papers by Brown & Michael (1954), Mangler & Smith (1959) and Smith (1968). Brown & Michael replaced the spiral vortex system by an isolated vortex and cut. Mangler & Smith improved this model by introducing a finite vortex sheet springing from the leading edge. This model was developed further by Smith.

T. A. Diab & M. Judd extended the basic Brown & Michael model by condensing the vorticity associated with the spiral sheet onto several line vortices joined by cuts. For the case of a flat delta wing the results showed no consistent improvement upon the original single-vortex model, although it is intended to extend the calculations to wings with thickness and anhedral. J. E. Barsby, who used the model of Smith (1968), showed that when a flat-plate delta wing is placed at a sufficiently low angle of incidence, with the Kutta condition imposed at the leading edge, the flow separates not at the leading edge but at a small distance inboard. Such vortices are very small and have not been observed experimentally. Results for a slender delta wing at angles of incidence and yaw were presented by I. P. Jones. For sufficiently large angles of yaw the leeward leading edge becomes a trailing edge and in this situation he was able to adopt a linear model for the wing–trailing-vortex combination. The resulting analytical solution showed good agreement with calculations based upon the nonlinear model of Smith (1968).

In the papers described above, the authors incorporated a conical-flow approximation. J. H. B. Smith demonstrated the existence of a family of non-conical self-similar solutions for slender wings with an appropriate planform shape and lengthwise camber. He showed that an approximate solution obtained by Legendre (1952) for a delta wing is in fact exact for a different planform. A calculation procedure for a more general combination of planform and lengthwise camber was presented by R. W. Clark. Results were shown by him for a plane wing with a gothic planform and a delta wing with a parabolic lengthwise camber. For the latter case a marked deformation of the sheet close to the leading edge occurred somewhat ahead of the station at which the local angle of incidence vanishes. A quite different calculation procedure was employed by C. Rehbach. The method, which is not restricted to slender bodies, replaces the lifting surface by a vortex-lattice system, from which line vortices are allowed to extend into the flow, as in Rehbach (1971). To obtain a solution for a delta wing, he starts with a solution for a rectangular wing with side-edge separation. This planform is iteratively deformed to the configuration of interest, with separation from leading and trailing edges. The author is able to include subsonic trailing-edge effects.

Two local solutions appropriate to conical slender-body flows of this type were discussed. G. J. Clapworthy presented a complicated local expansion of the flow behaviour in the neighbourhood of a leading edge from which the vortex is shed. The analysis applies to wings with both zero and non-zero wedge angles at the leading edge. By contrast, E. C. Maskell considered the asymptotic spiral structure of the vortex sheet, producing a correction to the second-order term he derived earlier (see Küchemann 1965). The results obtained are in good agreement with the numerical work of Smith (1968).

J. Rom, H. Portnoy & C. Zorea described a numerical investigation of the rolling up of the wing trailing-vortex system. They represented this by discrete vortices for various linear and nonlinear lifting-surface models. The results showed good agreement with measured overall characteristics. In a similar way, B. Maskew represented the lifting surface itself by quadrilateral vortex rings and the trailing-vortex sheets by arrays of discrete vortices. Results were given for a rectangular wing of small aspect ratio with tip-edge separation, a wing with a flap and a tailplane, and a wing with a flap in a wind tunnel. D. Wheeler described an experimental technique to investigate the structure of a trailing vortex. This involves the introduction of polystyrene pellets into the flow field, and subsequent analysis of cinematic photographs of the motion. O. Svidén & W.

Kasper put forward some ideas for high-lift systems involving vortex motions. They showed a film, apparently demonstrating the possibility of controlled flight at extremely low forward speed with an extensive region of vortex flow over the wing.

Three papers were presented on unsteady vortex-sheet formation. D. W. Moore studied the evolution of a two-dimensional vortex sheet from a plane initial position. The sheet is replaced by a distribution of line vortices. The author gave an explanation for the chaotic behaviour of these vortices found by previous workers (e.g. Kuwahara & Takami 1973) who tried to repeat Westwater's (1936) calculations, and advanced a method for obtaining smooth roll-up. G. J. Hancock addressed himself to the problem of the Kutta condition in unsteady, inviscid, incompressible flow. In addition to insisting that the loading at the trailing edge is zero, Hancock argued that a further condition, that flow separation occurs there, must also be imposed. To satisfy the latter condition in his numerical calculations, he allowed the inclination of the shed sheet to be arbitrary. His results agree closely with Maskell's (1972) conclusion that the flow must be tangential to one or other surface of the aerofoil. In a lively discussion, it was suggested that an unnecessary degree of freedom might have been introduced into the calculations. For unsteady separated flow over slender delta wings, R. K. Cooper extended the leading-edge model of Smith (1968). Results were compared with experiment for the lift and moment due to pitching and heaving oscillations of a delta wing. The defects of the slender-body treatment were apparent.

3. Stability and decay of vortices

The advent of large passenger transport aircraft, with their vigorous trailing-vortex wakes, has been responsible for an upsurge of interest in the stability of such systems. This subject was introduced by S. E. Widnall, who investigated the self-induced motion and stability of vortex filaments containing an arbitrary distribution of swirl and possibly axial velocities. This study provides a rational basis for the 'cut-off' model of Crow (1970). The problems discussed included that of the stability of a vortex line containing an axial jet and the mutual inductance instability of a vortex pair. It was suggested in discussion that, since stratification of the fluid is ignored, caution should perhaps be exercised when comparing the results with the observed physical phenomena (e.g. see Scorer 1958, chap. 6). The problem of the stability of a single jet-vortex was pursued by S. C. Kot, who extended and reinterpreted the stability boundary of Krishnamoorthy (1966), reconciling it with the result for the pure jet. P. C. Parks & S. C. Kot also considered the stability of the twin vortex configuration with axial flows. It was noted in discussion that the results are at variance with Crow's cut-off theory, which should be approached in the long-wave limit. Motivated by the instability observed by Hawthorne in a curved pipe, R. S. Scorer carried out a local analysis in which the fluid motion was assumed to be part of a helical motion. He showed that, when the density is uniform, a vortex with helical particle paths is unstable for rotation of the fluid elements about directions which lie between the vorticity vector and the direction of the axis of the motion.

P. G. Saffman described a theory to explain the observed dependence on Reynolds number of the decay of turbulent line vortices. He showed that, independently of the distribution of Reynolds stress, the circulation in the outer part of the vortex develops an overshoot. The overshoot can give rise to a Taylor-vortex type of instability and this was tentatively advanced as an explanation for the 'fuzziness' observed in aircraft vapour trails. S. M. Damms & D. Küchemann presented a theoretical model to explain the two-dimensional mixing flow downstream of a splitter plate between two parallel streams of different speeds. In the model the streams are separated by a vortex sheet rolled up into an array of double-branched cores which are constrained to move on a straight line with the mean of the free-stream speeds. Preliminary results appear to be consistent with experimental evidence which indicates that some of the cores grow in a wedge whilst the others disappear. Experimental investigations of the decay of counter-rotating vortex pairs were described by H. Bippes. Remarkably, it was the flow outside the vortex cores which exhibited the first signs of instability. He concluded that the breakdown of Taylor–Görtler vortices in the boundary layer on a curved wall which leads to turbulence is related to the breakdown of the vortices which he observed behind a wing in a towing tank.

P. Bellamy-Knights showed a film of a nearly axisymmetric vortex-breakdown bubble in a cylindrical tube. Using flow visualization techniques, he deduced that the core filament proceeds along the axis in the vortex bubble in the stream direction but that elsewhere within the bubble there is reversal of the axial flow. The axial flow with swirl in a circular tube was also considered by P.-A. Mackrodt; here the swirl was induced by rotating the pipe. From a stability analysis he concluded that very small rotation rates are sufficient to render this Poiseuille-type flow unstable, and suggested that the instability observed in classical Poiseuille flow may be due to swirl introduced from the reservoir. When a rotating flow emerges from a pipe orifice the resulting jet itself has swirl. The structure of such a jet with small swirl has been analysed by Görtler (1954) and Loitsianskii (1953), who perturbed the classical round-jet solution. M. B. Glauert argued that if the swirl is sufficiently large the jet cannot remain concentrated along the axis. He presented a similarity solution in which the jet forms a thin layer whose mean surface is a hyperboloid of revolution.

The increasing problem of noise associated with jet aircraft has stimulated studies of the effects of sound on flows with vorticity. D. S. Jones & J. D. Morgan extended their earlier theoretical studies of the effect of sound on the behaviour of a vortex sheet separating two fluids in relative motion by introducing a model to represent the turbulent portion of the shear layer. The resulting modification to the sound pattern is thought to be relevant to the enhanced forward noise propagation and to a feedback mechanism for jet screech. In another theoretical contribution P. Cannell determined the low-Mach-number symmetric sound field generated by the motion of two line-vortex filaments coupled to a two-dimensional semi-infinite duct. This adds to the small number of analytic solutions which can be used to support such general treatments as those of Curle (1955) and

Powell (1964). The wave motion produced in a turbulent jet by an imposed transverse sound field was described by D. Bechert. In his experiments the observed vortex roll-up was very similar to the laminar flow case. A parabolic jet-wave envelope was deduced from a semi-empirical theory for small deflexions of the jet and low Strouhal numbers. E. Pfizenmaier & D. Bechert devized ingenious experiments, involving the synchronization of an oscillating mirror with the flow unsteadiness, to determine how a shear layer is shed from a salient edge in a high-frequency unsteady flow. They deduced that the Kutta condition was not satisfied at their Reynolds numbers of 10^4–10^5. They stressed that this was not sufficient to resolve the question of the Kutta condition in an inviscid flow model.

4. Atmospheric vortices

Vortex motions in the atmosphere occur on a much larger scale than those described hitherto. They are less well understood at this time but are increasingly attracting the attention of fluid dynamicists and meteorologists.

Several workers (see for example Serrin 1972) have modelled atmospheric tornadoes but L. Hatton confined his attention to the considerably less destructive waterspout. The boundary condition which he applied at the water surface is one of non-zero stress combined with a slip velocity, although the implications of this for the motion below the surface were not considered. The model predicted upflow of greatest magnitude in an annular region outside the core in accordance with visual observations. L. Bode & L. M. Leslie presented preliminary results from numerical experiments associated with a vortex generated by forced convection in a rotating environment. Their aim is to extend the work of Leslie (1971) by investigating the effects of constraints which various physical and computational boundary conditions impose upon the flow. R. Kh. Zeytounian described his work on three problems involving vortex formation in a perfect fluid: lee waves behind obstacles in a stratified fluid, shear flow past an obstacle in a channel, and flow through the blade rows of an axial turbine.

5. Vortex rings

Vortex rings are a familiar physical phenomenon. E. A. Boyd, B. E. A. Jacobs & P. van der Meer suggested that the persistent nature of vortex rings may be exploited to reduce the pollution level in an industrial environment by pulsing the discharge from a chimney. Preliminary results from their experiments were shown, together with results by O. Arnold from Karlsruhe. The latter appeared to show thin vortex rings, but it was noted that the method of marking them with dye lines did not show their true structure, which was revealed more clearly by their measured deceleration. The decay of these relatively fat vortex rings has been explained by Maxworthy (1972). Vortex rings were also the subject of a contribution by P. O. A. L. Davies, who was concerned with the initial development of a circular jet. In computer calculations the axisymmetric shear layer formed at an orifice was modelled by a succession of elementary circular vortices. These formed clusters which Davies identified with the vortex rings observed in shock-tube experiments. D. Baxter used flow visualization techniques in air and water to investigate the convection and development of vortex rings. He deduced that the vortex development consists of a sequence of well-defined stages. It was suggested that the theory developed by S. E. Widnall might explain observed asymmetries.

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